Quadratic Functions in Standard Form

In Example 4 we saw that the function $f(x) = 3(x - 4)^2 - 1$ can be expressed in the general form of a quadratic function. This is an example of a quadratic function in standard form according to the following definition.

Standard Form of a Quadratic Function

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square.

The next example shows how "completing the square" allows us to express a quadratic function in standard form.

example 5 Expressing Quadratic Functions in Standard Form

Express the quadratic function $f(x) = x^2 + 16x + 24$ in standard form.

Solution

To get f in standard form we "complete the square" by first taking half of the coefficient of x and squaring it: $\left(\frac{16}{2}\right)^2 = 64$. We then add and subtract the result.

$$\begin{aligned} f(x) &= x^2 + 16x + 24 && \text{Given function} \\ &= \underbrace{(x^2 + 16x + 64)}_{\text{perfect square}} - 64 + 24 && \text{Complete the square: Add 64 inside parentheses, and subtract 64 outside} \\ &= (x + 8)^2 - 40 && \text{Factor and simplify} \end{aligned}$$

We study how to factor perfect squares in Algebra Toolkit B.2, page T33.

The standard form is $f(x) = (x + 8)^2 - 40$.

■ **NOW TRY EXERCISE 21** ■

example 6 Expressing a Quadratic Function in Standard Form

Express the quadratic function $f(x) = 2x^2 - 12x + 23$ in standard form.

Solution

Since the coefficient of x^2 is not 1, we must factor this coefficient from the terms involving x *before* completing the square.

$$\begin{aligned} f(x) &= 2x^2 - 12x + 23 && \text{Given function} \\ &= 2(x^2 - 6x) + 23 && \text{Factor 2 from } x\text{-terms} \\ &= 2\underbrace{(x^2 - 6x + 9)}_{\text{perfect square}} - 2 \cdot 9 + 23 && \text{Complete the square: Add 9 inside parentheses, and subtract } 2 \cdot 9 \text{ outside} \\ &= 2(x - 3)^2 + 5 && \text{Factor and simplify} \end{aligned}$$

The standard form is $f(x) = 2(x - 3)^2 + 5$.

■ **NOW TRY EXERCISE 23** ■